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## Polymeric Fluid Flow Obeying ECTN Model in an Inclined Circular Tube with Permeable Wall

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### Abstract

In this paper, the flow of a polymeric fluid obeying ECTN (Energetically Cross linked Transient Network)model in an inclined circular tube with permeable wall is analyzed. In this study, the rates of segment creation and loss formed by physical and energetic interactions are assumed to be constants. The expressions for the velocity field and the flow rate are obtained. The results are evaluated numerically for various values of physical parameters Nm and . It is found that the flow shows down axially as against Newtonian flow in a circular tube. The flow analysis presented for polymeric fluids has applications in understanding the characteristics of semi-dilute and concentrated solutions of hydrophilic polymers.

**Keywords**: Polymeric flow, Permeable wall, modified Reynolds number Nm,  ECTN model.

## Introduction

Several diverse and seemingly unusual rheological phenomena such as double stress overshoots, maximum in elongation viscosity with stretch rates, phase separation in deforming solutions, uncharacteristically long restoration times, etc., is observed in solutions of polar polymers. These polymers might have a common physical origin, which is related to shear-induced modification of the rates of creation and loss of energetic transient cross links (hydrogen bonds). It is reported that the dynamics of energetic cross links (H-bonds) are fundamentally different than those of physical cross links (chain entanglements). Some of these essential differences are incorporated in a phenomenological framework of a transient network theory namely the Energetically Cross linked Transient Network (ECTN) model. The ECTN model can successfully predict double stress overshoots in shear flows and the maximum in elongation viscosity with stretch rates in elongation flows.

Biofluid is a fluid that flows in a physiological system. Mucus, saliva, blood, water etc., are some of the well-known biofluids in nature. The medicines prescribed by Doctors in the treatment of patients contain polymer solutions, which also come under artificial biofluid category.

Seright et al. [1] & [2]. Delshad et al. [3] and Seright [4] reported that some polymer solutions may exhibit pseudodilatant behavior in porous media. These studies have shown that understanding and modeling of polymer rheology in porous media are also important. This study also has tried to capture the effects of non-Newtonian fluid on the sweep, oil recovery, and injectivity in the heterogeneous multilayered reservoir.

This apart, investigations on polymer solution have important applications in biology and medicine. Some physiological systems such as lung alveolar sheet are modeled as channels/tubes with permeable walls covered by porous media (see Tang and Fung, [5]). The fluid in the physiological systems is assumed to be either Newtonian or non-Newtonian depending on the physical situation (vide Shukla et al. [6], Chaturani and Ponnalagar swamy, [7]).

Energetically cross linked transient network (ECTN) model is used in the present work as developed by Lele and Mashelkar [8]. This model is based on the assumption that the dynamic of energetic cross links ($H$-bonds) are fundamentally different them those of physical cross links (chain entanglement) and polymer solution consists of two types $I$ = P (Physical) and E(Energetic) of inter-penetrating networks formed by physical and energetic interactions which account for the rates of segment creation and loss.

In view of several applications, it will be interesting to study the polymer fluid (obeying ECTN model) flow in a conduit of circular cross section. In the polymeric shear flow we assume that thermal variations

are small and do not contribute to any segment order of change in the energy of the flow. Thus, the problem is confined to an isothermal steady shear flow through a pipe arising from constant pressure gradient. The effect of various physical parameters on the flow quantities are discussed through graphs.

## Stress Tensor for ECTN Model

Let L be the rate of creation of segments of length $Q$ at time $t$ and let $\lambda_i^{-1}$ be the rate of destructions of segments of length $Q$. Let $f_i(\bar{t}) = L_r/L_{i0}$ and $g_i(\bar{t}) = \lambda_{i0}/\lambda_i$ be two non-dimensional parameters for rate of segment creation and destruction. Thus, the excess of creation over destruction is $[f_i(\bar{t})-g_i(\bar{t})]$. We define the co-efficient of growth over destruction as

$$Cr = \frac{f_i(\bar{t}) - g_i(\bar{t})}{g_i(\bar{t})} \qquad (1)$$

as a general coefficient valid for every $i$. The appropriate modeling of this parameter depends on the nature of $f(t)$ and $g(t)$ this defines the rheological state and the properties of the fluid. The contribution of the newly formed parameter $c_r$ is added to the Oldroyd model (see Bird et al. ( [9], [10] ). The modified stress equation is

$$\tau + \lambda_1 \tau_{(1)} + \frac{1}{2}\lambda_3\left\{\gamma_{(1)} : \tau + \tau : \gamma_{(1)}\right\} + \frac{1}{2}\lambda_5 (tr\tau)\gamma_{(1)} + \frac{1}{2}\lambda_6\left(\tau : \gamma_{(1)}\right)\delta$$

$$= -\eta_0\left[\gamma_{(1)} + \lambda_2\gamma_{(2)} + \lambda_4\left\{\gamma_{(1)} : \gamma_{(1)}\right\} + \frac{1}{2}\gamma_7\left(\gamma_{(1)} : \gamma_{(1)}\right)\delta\right] + \lambda_8 c_r \delta$$

$$(2)$$

In this equation, the parameter $\lambda_8$ has, dimensions of stress. In our studies we include this parameter along with the time parameter $\lambda_1$ to see the effect of $\lambda_8$ over $\lambda_1$. The other higher order effects are ignored. The model then simplifies to the form

$$\tau + \lambda_1 \tau_{(1)} = -\eta_0 \gamma_{(1)} + \lambda_8 C_r \delta \quad (3)$$

Equation (3) can be simplified using the following non-dimensionalization.

$$\tau = \sum_{i=P.E} G_{i0}\bar{\tau}_i, \quad \bar{\tau}_{i(1)} = \frac{\lambda_{i0}\tau_{i(1)}}{G_{i0}}, \quad \bar{\lambda}_8 = \frac{\lambda_8}{G_{10}} \ and \ \bar{\gamma}_{(1)} = \lambda_{i0}\gamma_{(1)}$$

$$(4)$$

with the fluid state parameter $\lambda_8$ of stress shall primarily be such that the term $\lambda_8 C_r \delta$ explains the thinning or thickening of the fluid. Thus, it can be assumed to be $G_{i0}$. This implies that $\lambda_8 = 1$.

$$g_i(\bar{t})\bar{\tau}_i + \bar{\tau}_{i(1)} = -\beta_i\bar{\gamma}_{(1)} + \left[f_i(\bar{t}) - g_i(\bar{t})\right]\bar{\delta}$$

$$(5)$$

where $f_i(\bar{t})$ and $g_i(\bar{t})$ are respectively, the rates of segment creation and loss for $i = P, E$ Following, Lele and Masheklar [8]

$$f_P(\bar{t}) = 1 , \qquad g_P(\bar{t}) = 1$$

$$(6)$$

$$f_E(\bar{t}) = c_1, \qquad g_E(\bar{t}) = c_2$$

$$(7)$$

## Non Dimensionalization of the Governing Equations

Let $\rho, V, R$, β, α are the average density, velocity of the fluid, the radius of the circular cylinder, permeability parameter and angle of inclination. These parameters are primitive variables for any experimental studies and form the trio for non – Dimensionalization of the motion equation. Thus, we write.

$$\bar{v}^* = \frac{1}{V}v^*, \quad \bar{t} = \frac{V}{R}t, \quad \bar{p} = \frac{1}{\rho V^2}p, \quad \bar{r} = \frac{r}{R}, \quad \bar{\nabla} = R\nabla, \quad \bar{F}^* = \frac{R}{\rho V^2}F^*$$

$$(8)$$

Ignored asterisks, the governing equations can be written as

(a) Equation of Continuity

$$\nabla.v = 0 \qquad (9)$$

(b) Equation of Motion

$$\frac{\partial(\rho v)}{\partial t} = -[\nabla.(\rho v)v] - [\nabla.\pi] + F \qquad (10)$$

Where $\pi = p\delta + \tau$, $p$ is an isotropic pressure.

$$\rho\frac{Dv}{Dt} = -\nabla p - [\nabla.\tau] + F \qquad (11)$$

Using non – dimensional parameters, defined in (5) and (8), the above equation is written as

$$\frac{Dv}{Dt} = -\nabla p - \left[\nabla.\left\{\sum_i (\tau)_i \frac{G_{i0}}{\rho V^2}\right\}\right] + F \ , \qquad where$$

i=P,E.   (12)

The above equation is written as

$$\frac{Dv}{Dt} = -\nabla p - \frac{G_{p0}}{\rho V^2}\left[\nabla.(\tau)_P + \frac{G_{E0}}{G_{P0}}\nabla.(\tau)_E\right] + F$$

$$(13)$$

We know that Reynolds number $\mathrm{Re} = \dfrac{\rho VR}{\eta_0}$ and in our

context it takes the form

$$\mathrm{Re} = \frac{\rho V^2}{G_{P0}} \text{ where } \eta_0 = \frac{RG_{P0}}{V} \quad (14)$$

We define network ratio of creation and destruction of segments as $K = \dfrac{G_{E0}}{G_{P0}}$. Thus the equation of motion is

$$\frac{Dv}{Dt} = -\nabla p - \frac{1}{\mathrm{Re}}\left[\nabla.(\tau)_P + K\nabla.(\tau)_E\right] + F \quad (15)$$

Where F is the body force

## Formulation of the Problem

The fully developed steady laminar flow of polymeric fluid (obeying ECTN model) flow through a long in an inclined circular tube with permeable wall is analyzed. Here the circular tube is of radius R, length L inclined at angle α with the horizontal. The flow is maintained by the presence of a constant pressure gradient along the axis of the tube. The body forces are neglected, cylindrical co-ordinate system is used. Let the axis of the tube be taken as z-axis along which the flow takes place and r denote the radial distance measure from perpendicular to the z-axis.
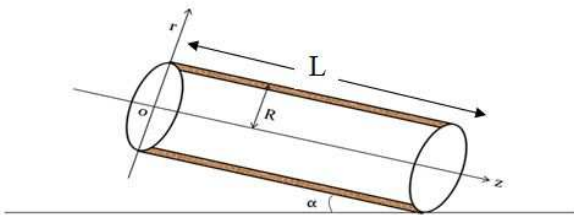


**Fig 1: Physical model**

The pressure at the pipe ends at $z = 0$, $z = L$ are $p_1$ and $p_2$. Thus, the average constant pressure gradient is assumed to be $\dfrac{dp}{dz} = \dfrac{p_2 - p_1}{L}$. In non-dimensional form, this becomes $\dfrac{d\overline{p}}{dz} = \overline{p}_2 - \overline{p}_1 = \alpha_0$ and for flow to be along positive z-axis, $\alpha_0 < 0$. Since pressure gradient is along z-axis, only non – zero component of velocity is $\overline{v}_z$. Due to axial symmetry, state and space variables are independent of $\theta$. Thus, we postulate that

## Governing Equations

$$v_z = v_z(r), \quad v_r = 0, \quad v_\theta = 0, \quad \frac{\partial}{\partial \theta}() = 0$$
(16)

The continuity equation (8) and equation of motion (15) in cylindrical coordinates reduce to (Vide Mujumdar and Sharma [11]) :

a) Continuity Equation
$$\frac{\partial v_z}{\partial z} = 0$$
(17)

b) Equations of Motion
$$0 = -\frac{\partial p}{\partial z} + \frac{\sin\alpha}{f} - \frac{1}{\mathrm{Re}}\frac{1}{r}\left[\frac{\partial}{\partial r}\left\{r(\tau_{rz})_P + K(\tau_{rz})E\right\}\right]$$
(18)

$$0 = -\frac{\partial p}{\partial r} - \frac{1}{\mathrm{Re}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r(\tau_{rr})_P + K(\tau_{rr})E\right\}\right]$$
$$-\frac{1}{r}(\overline{\tau}_{\theta\theta})_P + K(\overline{\tau}_{\theta\theta})_E \quad (19)$$

$$0 = -\frac{1}{r}\frac{\partial p}{\partial z}$$
(20)

## Simplification of Stresses

Equations of stress components as derived from constitutive equation (5) for two dimensions in cylindrical coordinates for steady state turn out to be

$$g_i(t)(\tau_{zz})_i = 2\left(\frac{\partial v_t}{\partial r}\right)(\tau_{rz}) + \left[f_i(t) - g_i(t)\right]$$

$$g_i(t)(\tau_{zz})_i = \left(\frac{\partial v_t}{\partial r}\right)(\tau_{rr})_i - \beta_i\frac{\partial v_z}{\partial r}$$

$$g_i(t)(\tau_{rr})_i = \left[f_i(t) - g_i(t)\right];$$

$$g_i(t)(\tau_{\theta\theta})_i = \left[f_i(t) - g_i(t)\right]$$

where $f_i(t)$ and $g_i(t)$ are constant functions as given in equations (6) and (8).

Using equations (6) and (8) the above equations simplify to

$$(\tau_{rr})_p = 0, \qquad (\tau_{rz})_p = \beta_P \frac{dv_z}{dr}, \qquad (\tau_{zz})_p = -2\beta_P \left(\frac{dv_z}{dr}\right)^2,$$

$$(\tau_{\theta\theta})_p = 0, \qquad (\tau_{rr})_E = \frac{c_1 - c_2}{c_2}, \qquad (\tau_{rz})_E = -\left(\frac{c_1 - c_2}{c_2^2} + \frac{\beta_E}{c_2}\right)\frac{dv_z}{dr},$$

$$(\tau_{zz})_E = 2\left(\frac{c_1 - c_2}{c_2^3} + \frac{\beta_E}{c_2^2}\right)\left(\frac{dv_z}{dr}\right) + \frac{c_1 - c_2}{c_2}, \qquad (\tau_{rr})_E = \frac{c_1 - c_2}{c_2},$$

(21)

### Formulation of Model Equations

On substituting the above expressions of stresses in (18) and (19), we obtain the basic equations in the following form :

$$\frac{\partial p}{\partial z} - \frac{\sin\alpha}{f} = -\frac{1}{\mathrm{Re}}\frac{1}{r}\left\{\frac{d}{dr}\left[r\left(-\beta_P - K\left(\frac{c_1 - c_2}{c_2^2} + \frac{\beta_E}{c_2}\right)\frac{dv_z}{dr}\right)\right]\right\}$$

(22)

$$\frac{\partial p}{\partial r} = -\frac{1}{\mathrm{Re}}\left\{\frac{1}{r}\frac{d}{dr}\left[r K\left(\frac{c_1 - c_2}{c_2}\right)\right] - K\frac{1}{r}\left(\frac{c_1 - c_2}{c_2}\right)\right\}$$

(23)

### 5. Solutions of the Problem

Equation (23), simplifies to $\frac{\partial p}{\partial r} = 0$. This gives us $p = F(z)$ and $\frac{\partial p}{\partial z} = F'(z)$ but $\frac{\partial p}{\partial z} = \alpha_0$ it follows that $p = \alpha_0 z$. Now equation (22) can be written as

$$\alpha_0 = \frac{\sin\alpha}{f} + \frac{1}{\mathrm{Re}}\left\{\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)\right]\left[\beta_P + K\left(-\frac{c_1 - c_2}{c_2^2} + \frac{\beta_E}{c_2}\right)\right]\right\}$$

(24)

Boundary conditions to be satisfied are

$$\frac{dv_z}{dr} = 0, \qquad\qquad r = 0$$

$$v_z = -\beta\frac{dv_z}{dr}, \qquad r = 1$$

(25)

where β is the Permeability parameter

We introduce a modified Reynolds number Nm as

$$Nm = \frac{\mathrm{Re}}{\left[\beta_P + K\left(-\frac{c_1 - c_2}{c_2^2} + \frac{\beta_E}{c_2}\right)\right]}$$

(26)

The equation of motion (24) thus simplified as

$$\alpha_0 = \frac{\sin\alpha}{f} + \frac{1}{Nm}\frac{1}{r}\left[\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)\right]$$

(27)

The solution is

$$v_z = \left(\frac{Nm\,\alpha_0}{4} - \frac{\sin\alpha\;Nm}{4f}\right)r^2 + A_1$$

$$where\; A_1 = \frac{Nm}{4}\left\{2\beta\left[\alpha_0 + \frac{\sin\alpha}{f}\right] + \left[\frac{\sin\alpha}{f} - \alpha_0\right]\right\}$$

(28)

### Deductions and Discussions

**a)** As α → 0, Nm → 1, the result (28) reduces to corresponding results of Hagen-Poiseuille flow of a Newtonian fluid in a circular pipe.

**b) Stress distribution:**

Using solution (28) and assuming stretch rate $\beta_i = 1$, we get

$$\tau_{rz} = -\left\{\beta_P + K\left[-\frac{c_1 - c_2}{c_2^2} + \frac{\beta_E}{c_2}\right]\right\}\frac{\partial v_z}{\partial r}, \qquad \tau_{\theta\theta} = K\left(\frac{c_1 - c_2}{c_2}\right),$$

$$\tau_{rr} = K\left(\frac{c_1 - c_2}{c_2}\right), \qquad \tau_{zz} = -2\left\{\beta_P + K\left(-\frac{c_1 - c_2}{c_2^3} + \frac{\beta_E}{c_2^2}\right)\right\}\left(\frac{\partial v_z}{\partial r}\right)^2 + K\frac{c_1 - c_2}{c_2},$$

(29)

The steady shear material function can be derived as the shear rate dependent viscosity and first normal stresses difference at any point are given by

$$\overline{\tau}_{rz} = \eta\frac{d\overline{v}_z}{dr}, \qquad\qquad where\; \eta = -Nm$$

$$(\overline{\tau}_{zz}) - (\overline{\tau}_{rr}) = \psi_1\left(\frac{d\overline{v}_z}{dr}\right)^2$$

where $\psi_1 = -2\left\{\beta_P + K\left[-\left(\frac{c_1 - c_2}{c_2^3}\right) + \frac{\beta_E}{c_2^2}\right]\right\}$

(30)

**c) Analysis of Parameter Nm**

For the parameter Nm defined by (26), it is worthwhile to discuss the modified viscosity $\eta_m$. This modified parameter $\eta_m$, in this case is $Nm = \rho RV / \eta_m$ and we know that $\mathrm{Re} = \rho RV / \eta_0$. Hence, we obtain

where

$$\eta_m = \eta_0 \left[ \beta_p + K \left( -\frac{c_2 - c_1}{c_2^2} + \frac{\beta_E}{c_2} \right) \right]$$

(31)

It is thus seen that the Reynolds number for ordinary fluid is greater than the corresponding number modified Reynolds number Nm of the same fluid added with polymeric matters.

**d) Maximum velocity**

The maximum velocity occurs on the axis of the pipe and is given by

$$(v_z)_{max} = \frac{Nm\alpha_0}{2} - \frac{(Nm)\sin\alpha}{2f}$$

(32)

## Conclusions

In this paper, analytical expressions are derived for velocity vector, the stress components and viscosity function in a fully developed cylindrical pipe flow using energetically cross linked transient network (ECTN) model. The analysis clearly shows that the velocity profile depends on the parameters Nm, $\alpha_0$ (average constant pressure gradient), $\beta$ (permeability parameter), R(radius), $v$ (velocity) and Re(Reynolds number). The flow shows down axially as against Newtonian flow in a cylindrical pipe.

The variation of velocity profile ($\bar{v}_z$) varies as $\bar{r}$ is calculated, for different values of $\alpha_0$ and $\beta$ and is shown in Figures 2 and 3 for fixed $v$ =0.3and R=0.02. We observe that the velocity increases with increasing $\alpha_0$ or $\beta$.

The variation of velocity profile ($\bar{v}_z$) varies as $\bar{r}$ is calculated, for different values of radius R and is shown in Fig.4, for fixed $\alpha_0$=-0.01, $v$ =0.3and $\beta$ =0.1. We observe that the velocity increases with the decrease in radius R.

The variation of velocity profile ($\bar{v}_z$) varies as $\bar{r}$ is calculated, for different values of Re, Nm and $\alpha$ and is shown in Figures 5, 6 and 7 for fixed $\alpha_0$=-0.01, $v$ =0.3, $\beta$=0.1 and R=0.02. We observe that the velocity increases with increasing Re or Nm or $\alpha$.
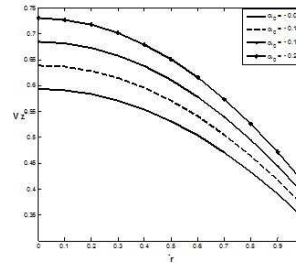


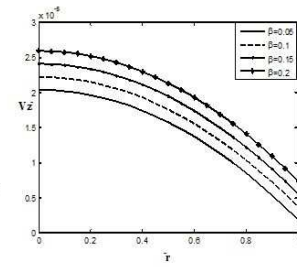**Fig. 2:** Velocity profile for different values of constant pressure $\alpha_0$gradient.

**Fig. 3:** Velocity profile for different values of Permeable parameter $\beta$
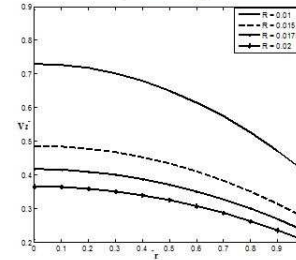
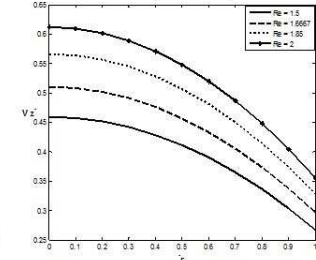**Fig. 4:** Velocity profile for different values of Radius 'R'.

**Fig 5:** Velocity profile for different value Reynolds number 'Re'.

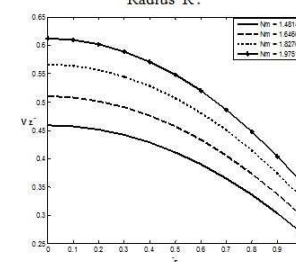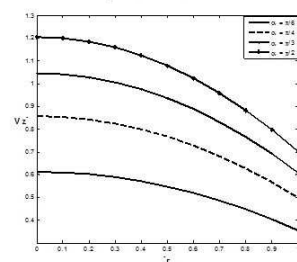**Fig. 6:** Velocity profile for different values of Modified Reynolds number 'Nm'.

**Fig.7:** Velocity profile for different values of inclination $\alpha$.

## Acknowledgements

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